University of Tripoli Faculty of Engineering Electrical and Electronic Department
Spring 2015 Final Exam **Signals & System (EE302)** Date: 25/06/2015 Time: 3Hrs
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Q3)

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + b_k \sin k\omega_0 t$$

$$T_0 = 2\pi, \qquad \omega_0 = 1$$

$$a_k = \frac{2}{2\pi}\int_0^{2\pi} (\frac{-t}{2\pi} + 1) cos\, ktdt = \frac{-1}{2\pi^2}\int_0^{2\pi} tcos\, ktdt = 0$$

$$b_k = \frac{2}{2\pi}\int_0^{2\pi} (\frac{-t}{2\pi} + 1) sin\, ktdt = \frac{-1}{2\pi^2}\int_0^{2\pi} tsin\, ktdt = \frac{1}{2\pi^2}\left[\frac{1}{k^2}(-kt)sinkt\right]_0^{2\pi} = \frac{1}{k\pi}$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} (\frac{-t}{2\pi} + 1)\, dt = \frac{1}{2\pi}\left[\frac{-t^2}{4\pi} + t\right]_0^{2\pi} = \frac{1}{2}$$

$$x(t) = \frac{1}{2} + \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{1}{k} \sin kt$$

a)

$$D_0 = \frac{1}{2\pi}\int_0^{2\pi} (\frac{-t}{2\pi} + 1)\, dt = \frac{1}{2\pi}\left[\frac{-t^2}{4\pi} + t\right]_0^{2\pi} = \frac{1}{2}$$

$$D_k = \frac{1}{2\pi}\int_0^{2\pi} (\frac{-t}{2\pi} + 1)e^{-ikt}\, dt = \frac{-1}{4\pi^2}\int_0^{2\pi} te^{-ikt}\, dt + \frac{1}{2\pi}\int_0^{2\pi} e^{-ikt}\, dt$$

$$D_k = \frac{-1}{4\pi^2}\left[\frac{e^{-ikt}}{-k^2}(-ikt - 1)\right]_0^{2\pi} = \frac{-i}{2\pi k}$$

Q4)

a)

$$F(x(t - t_0)) = \int_{-\infty}^{\infty} x(t - t_0)e^{-i\omega t}dt$$

Let $\tau = t - t_0$

$$F(x(t - t_0)) = \int_{-\infty}^{\infty} x(\tau)e^{-i\omega(\tau + t_0)}d\tau = e^{-i\omega t_0}\int_{-\infty}^{\infty} x(\tau)e^{-i\omega\tau}d\tau = e^{-i\omega t_0}X(\omega)$$

$$\Rightarrow \quad x(t - t_0) \leftrightarrow e^{-i\omega t}X(\omega)$$

b)

The FT of the equation

$$j\omega Y(\omega) + 2Y(\omega) = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{2 + j\omega}$$

$$X(\omega) = \frac{1}{1 + j\omega}$$

$$Y(\omega) = X(\omega)H(\omega) = \frac{1}{(1+j\omega)(2+j\omega)} = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$$

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

Q5)

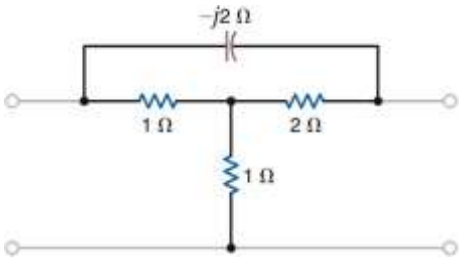
a)

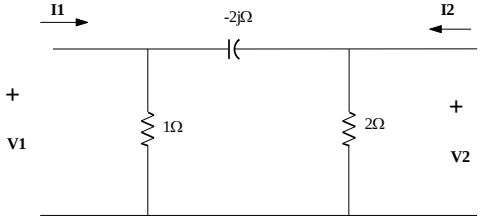
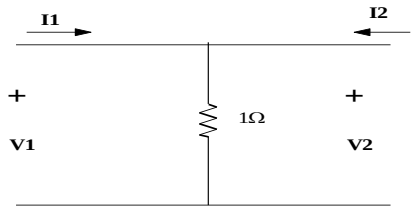
$$L(x(at)) = \int_{-\infty}^{\infty} x(at)e^{-st}dt$$

Let $\tau = at$

$$L(x(at)) = \frac{1}{a}\int_{-\infty}^{\infty} x(\tau)e^{-(\frac{s}{a})\tau}d\tau = \frac{1}{a}X(\frac{s}{a})$$

b)



	$Z_{11} = \frac{V_1}{I_1}\|_{I_{2=0}} = \frac{2-2i}{3-2i}$ $Z_{21} = \frac{V_2}{I_1}\|_{I_{2=0}} = \frac{2}{3-2i}$ $Z_{12} = \frac{V_1}{I_2}\|_{I_{1=0}} = \frac{2}{3-2i}$ $Z_{22} = \frac{V_2}{I_2}\|_{I_{1=0}} = \frac{2-4i}{3-2i}$	$Z_a = \begin{bmatrix} \frac{2-2i}{3-2i} & \frac{2}{3-2i} \\ \frac{2}{3-2i} & \frac{2-4i}{3-2i} \end{bmatrix}$
	$Z_{11} = \frac{V_1}{I_1}\|_{I_{2=0}} = 1$ $Z_{21} = \frac{V_2}{I_1}\|_{I_{2=0}} = 1$ $Z_{12} = \frac{V_1}{I_2}\|_{I_{1=0}} = 1$ $Z_{22} = \frac{V_2}{I_2}\|_{I_{1=0}} = 1$	$Z_b = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Thus for series network

$$Z = Z_a + Z_b = \begin{bmatrix} \frac{5-4j}{3-2j} & \frac{5-2j}{3-2j} \\ \frac{5-2j}{3-2j} & \frac{5-6j}{3-2j} \end{bmatrix}$$